Notes for homework assignment 2001.2.2

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1 Link utilisation U

As you have noticed, the homework assignment 2.2 was broken (as it was in 1999). Answers made according to 1999 model answer 6 are accepted, but I think it is good to make a comment on how this would have been better.

1.1 Values

B_D Number of bits in a data frame

B_A Number of bits in an ACK frame

C Link capacity [b/s]

T_T Timeout

P_b BER

l Length of the link

v Propagation speed of the signal in the medium

T_{cd} Time used to check a received data frame (before starting to send an ACK frame).

T_{ca} Time used to check a received ACK frame (before starting to send a data frame).

1.2 Delays

First, define propagation delay

$$T_P = \frac{l}{v}.$$

Time used to send a data frame

$$T_D = \frac{B_D}{C}.$$

Time used to send an ACK frame

$$T_A = \frac{B_A}{C}.$$

1.3 Probabilities

The given value was the bit error rate P_b. Assuming bit errors are independent (which is not often the case, since they can happen in bursts), the probability P_D that a data frame goes through correctly is

$$P_D = (1 - P_b)^{B_D}.$$

Correspondingly for the ACK frames

$$P_A = (1 - P_b)^{B_A}.$$

1.4 Cycles

The events happen in cycles. There are three different possible cycles.

- Cycle without errors:
  1. Data frame is sent (T_D)
  2. Frame propagates (T_P)
  3. Frame is checked (T_{cd})
  4. ACK frame is sent (T_A)
  5. Frame propagates (T_P)
  6. Frame is checked (T_{ca})
- Cycle with a corrupt data frame:
  1. Data frame is sent (T_D)
  2. Frame propagates (T_P)
  3. Frame is checked (T_{cd})
  4. Timeout happens. (Note that T_P and T_{cd} do not affect the length of this cycle, since $T_T > T_P + T_{cd}$.)
- Cycle with a corrupt ACK frame:
  1. Data frame is sent (T_D)
  2. Frame propagates (T_P)
  3. Frame is checked (T_{cd})
  4. ACK frame is sent (T_A)
  5. Frame propagates (T_P)
  6. Frame is checked (T_{ca})
  7. Timeout happens. (Note that T_P, T_{cd}, T_A, T_P and T_{ca} do not affect the length of this cycle.)

1.5 Shared medium

First, consider an error free transmission line. Then, cycles without errors follow each other. Utilisation is then the ratio of the time used to send to the medium and the total length of the cycle. Let T_C denote the total length of a correct cycle:

$$T_C = T_D + T_P + T_{cd} + T_A + T_P + T_{ca} = T_D + T_{cd} + T_A + T_{ca} + 2T_P.$$

Then,

$$U_s = \frac{T_D + T_A}{T_C}.$$

If $B_D \gg B_A$, $T_P \gg T_{cd}$ and $T_P \gg T_{ca}$,

$$U = \frac{T_D}{T_D + 2T_P}.$$

In the case when there are bit errors, also other types of cycles happen. The average share w of different types of cycles (when the length of the trace closes infinity) is defined by the probabilities P_D and P_A. The weights for different cycles are given as follows. First, the nominator:

$$\begin{aligned} D &= P_D P_A T_C + (1 - P_D)(T_D + T_T) + P_D(1 - P_A)(T_D + T_T) \\ &= P_D P_A T_C + (1 - P_D P_A)(T_D + T_T). \end{aligned}$$

- A fully correct cycle happens when both data and ACK frame are not corrupted. These events are independent, and thus $w_c = P_D P_A T_C / D$.
- A cycle with a corrupt data happens with probability $1 - P_D$. The fraction of time affected is different from the fully correct cycle, so that a correcting term is needed. Thus, the frame has the weight

$$w_d = \frac{(1 - P_D)(T_D + T_T)}{D}.$$

- A cycle with a corrupt ACK frame can only happen when the data frame is correct.

$$w_a = \frac{P_D(1 - P_A)(T_D + T_T)}{D}.$$

Knowing the weights w_c, w_a and w_d, the link utilisation with errors can be calculated as a weighted sum of the utilisations for different cycles. First, the utilisation for cycles with a corrupt data frame

$$U_d = \frac{T_D}{T_D + T_T},$$

and the utilisation for cycles with a corrupt ACK frame:

$$U_a = \frac{T_D + T_A}{T_D + T_T}.$$

Then, the utilisation of a link with errors is:

$$
\begin{aligned}
U_e &= P_D P_A \frac{T_C}{D} \frac{T_D + T_A}{T_C} + \frac{(1-P_D)(T_D+T_T)}{D} \frac{T_D}{T_D+T_T} + \frac{P_D(1-P_A)(T_D+T_T)}{D} \frac{T_D+T_A}{T_D+T_T} \\
&= P_D P_A \frac{T_D+T_A}{D} + (1-P_D)\frac{T_D}{D} + P_D(1-P_A)\frac{T_D+T_A}{D} = \frac{P_D(T_D+T_A)+(1-P_D)T_D}{D} \\
&= \frac{P_D T_A + T_D}{P_D P_A T_C + (1-P_D P_A)(T_D+T_T)}.
\end{aligned} \tag{1}
$$

If we make the assumptions that $B_D \gg B_A$, $T_P \gg T_{cd}$ and $T_P \gg T_{ca}$,

$$U_e = \frac{T_D}{P_D(T_D+2T_P)+(1-P_D)(T_D+T_T)}.$$

1.6 Unidirectional links

In unidirectional links, there is a dedicated line to both directions (e.g. fibers). Let us calculate the link utilisation as a mean of the utilisation of both of these links. For error free transmission:

$$U = \frac{1}{2}\left(\frac{T_D}{T_D+T_{cd}+T_A+T_{ca}+2T_P} + \frac{T_A}{T_D+T_{cd}+T_A+T_{ca}+2T_P}\right) = \frac{1}{2}U_s.$$

Similarly, for transmissions with errors, $U = U_e/2$.