

# Hierarchical routing for large networks

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# Clustering

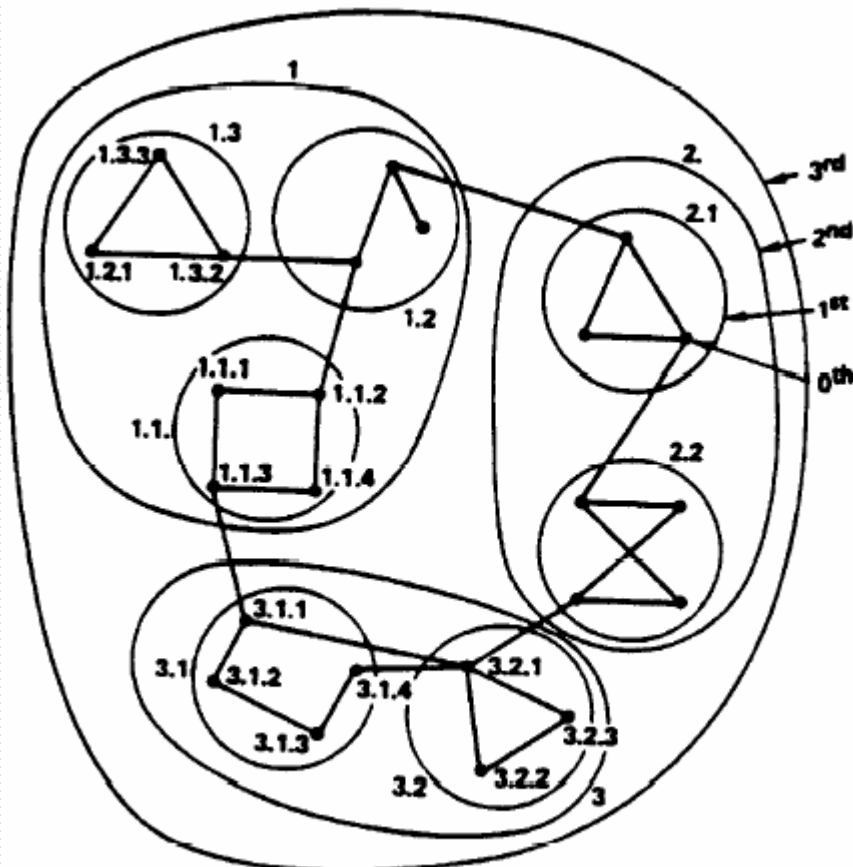


Fig. 1. A 3-level clustered 24-node network.

Routing table reduction:  
from 24 to 10

DESTINATION	NEXT NODE	DELAY	HOP NUMBER
NODES IN SAME CLUSTER	1.1.1		*
	1.1.2		
	1.1.3		
	1.1.4		
CLUSTERS IN SAME SUPERCLUSTER	1.1		*
	1.2		
	1.3		
SUPERCLUSTERS	1		*
	2		
	3		

\* = SELF ENTRY

Increase in path length:  
3.2.1 -> 3.1.4 by two hops

# Notation

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- $m$  – number of cluster levels
- $N$  – number of nodes in the network
- $l$  – routing table length
- $C_k$  –  $k$ th cluster
- $i_m$  –  $(m-1)$ th cluster to which  $C_k$  belong
- $n_k = \{n_k(i_{k+1})\}$  degree vector of  $k$ th cluster
- $n = (n_1, n_2, \dots, n_m)$  degree vector
- $h$  – path length in hops
- $d$  – diameter of the network

# Minimizing the routing table length

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- Real-value solution for fixed  $m$ 
  - $n_k(i_{k+1})=n_k=N^{1/m}, l = mN^{1/m}$
- Global optimal clustering
  - $m = \ln N, n_k=e=2.718..., l = e \ln N$
- Integer solution
  - Most two clusters of length 2, others 3
- No self-entries
  - $l=mN^{1/m}-m; n_k=2, m=\text{ceil}(\ln N/\ln 2)$

# Reduction of the routing table length

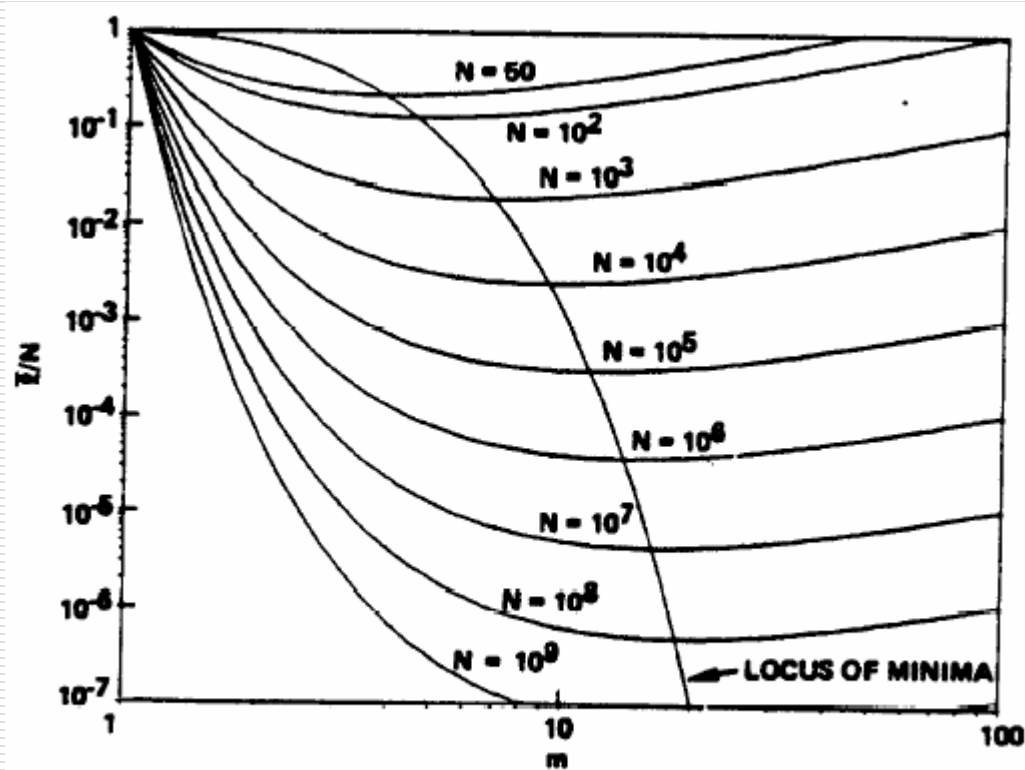


Fig. 4. Minimum relative table length,  $l/N$ , given  $m$ .

# Decrease in table length vs. increase in path length

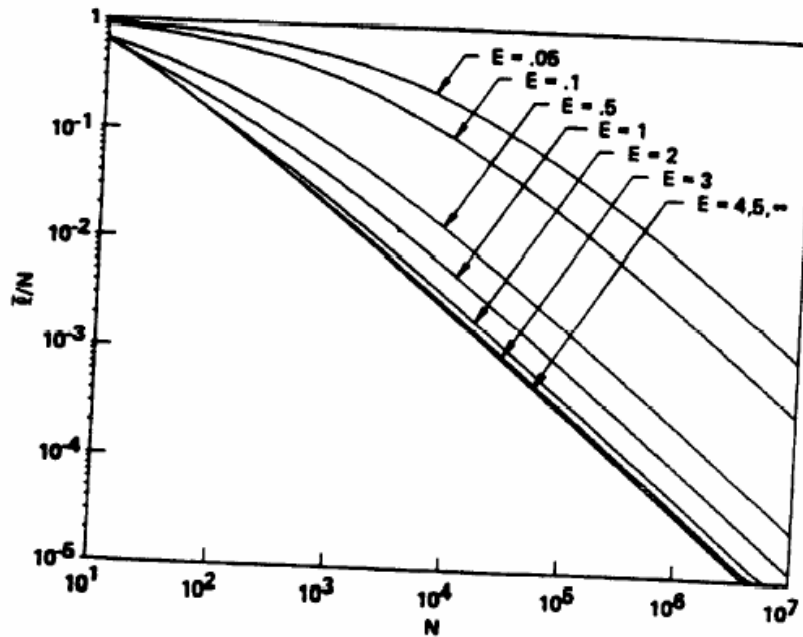


Fig. 10. Decrease in table length for a given maximum increase in path length.

*number of nodes, goes to infinity, the "static" performance of the mHR schemes approaches that of a non-clustered routing scheme, while the relative table length approaches zero; i.e.,*

$$N \rightarrow \infty \Rightarrow \begin{cases} h_c/h \rightarrow 1, \\ l/N \rightarrow 0. \end{cases}$$

# Increase in path length

- Closest Entry Routing (CER)
  - no data on intra-cluster routing to outside
- Overall Best Routing (OBR)
  - intra-cluster path lengths to outside
- Non-Cluster Routing (NCR)
  - degenerate case when clusters' length=1

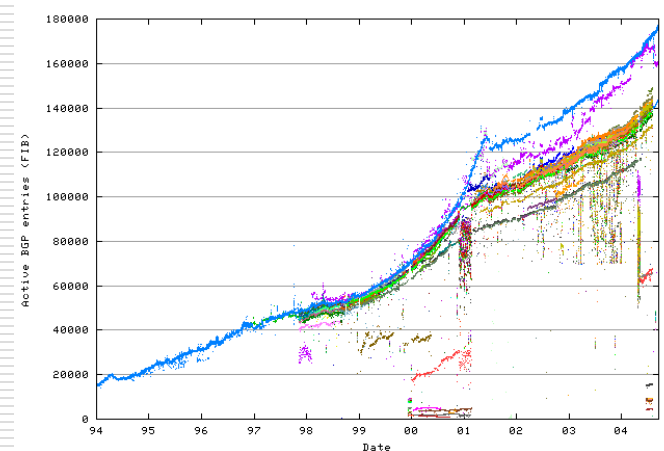
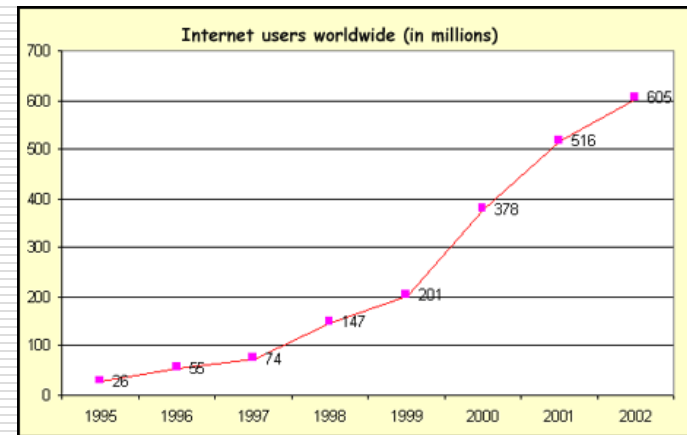
*Proposition 9. Under the conditions above and Assumption 3, the increase in the average path length in the network due to the reduction of routing information is such that*

$$h_c - h \leq \sum_{k=1}^{m-1} \left[ 1 - \frac{n_1 n_2 \dots n_k - 1}{N-1} \right] d_k. \quad (27)$$

# Internet

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- ❑ Three-level clustering
- ❑ Routing tables 200000 entries
- ❑ About 600 000 000 hosts
- ❑ Optimal 20 clusters, table length 50



# Conclusions

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- ❑ Hierarchical routing significantly reduces routing table size
- ❑ The side effect is increase of routing path
- ❑ Optimal clustering provides table length reduction from  $N$  to  $e \ln N$
- ❑ For large networks, the increase of path length is insignificant
- ❑ These ideas are applied in DHT design, and to some extent in the Internet