THE INTERPOLATED 3-D DIGITAL WAVEGUIDE MESH METHOD FOR ROOM ACOUSTIC SIMULATION AND AURALIZATION

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Abstract: In this paper we briefly overview the digital waveguide mesh method. It is a wave-based technique for room acoustic prediction operating in the time domain. The original technique suffers from direction dependent dispersion. In this paper we discuss couple of techniques that have been applied to overcome this problem. Firstly, the interpolated mesh structure, and secondly the frequency warping technique are presented. As a case study we present simulation results and visualizations obtained with two simple room geometries. In addition, the computational requirements of 3-D interpolated mesh are presented and possibility of real-time auralization is discussed.

Keywords: room acoustics, computer models, digital waveguide mesh

1 INTRODUCTION

The two main approaches for room acoustic modeling are the wave-based and the ray-based techniques. The ray-based methods are most suitable for high frequencies where the assumptions of geometrical room acoustics are valid. Nowadays these techniques are widely used in room acoustic design, and they can be used to predict room acoustic attributes quite reliably. However, for the modeling of lowest frequencies some wave-based methods are required. These techniques are essential if knowledge of eigenfrequencies of a room are needed.

In this paper we present one wave-based technique, the digital waveguide mesh. Originally the method was developed for physical modeling of 2-D musical instruments, but in this paper the main emphasis is on 3-D models that are most relevant from room acoustical viewpoint.

This paper is organized as follows. In Section 2 various digital waveguide mesh structures are presented and analyzed. In the next section the frequency warping technique and its application to the digital waveguide mesh is discussed. Two practical simulation examples and boundary conditions are elaborated in Sections 4 and 5, respectively. The computational complexity of the method is discussed in Section 6, and finally Section 7 concludes the paper.

2 DIGITAL WAVEGUIDE MESH

The digital waveguide mesh is a finite-difference time-domain (FDTD) technique, but its background is in digital signal processing. Originally the 2-D digital waveguide mesh was developed in 1993 for modeling of 2-D musical instruments such as membranes of drums [1]. In this section we present the 3-D mesh and certain improvements designed to overcome the inherent dispersion error in the original rectangular method.

2.1 Original 3-D Digital Waveguide Mesh

Digital waveguides are bi-directional unit-delay elements [2], and their first application was physical modeling of 1-D musical instruments such as strings of a guitar. The digital waveguide mesh consists of digital waveguides that are perpendicularly connected to each other forming a rectangular grid as illustrated in Fig. 1a such that in the 3-D case each node has a connection to six neighbors [3]. The equation to govern the behavior of a mesh is quite simple:

$$\begin{aligned} &p(n+1, x, y, z) \\ &= \frac{1}{3}\big[p(n, x+1, y, z) + p(n, x-1, y, z) + p(n, x, y+1, z) \\ &\quad + p(n, x, y-1, z) + p(n, x, y, z+1) + p(n, x, y, z-1)\big] \\ &\quad - p(n-1, x, y, z) \end{aligned} \quad (1)$$

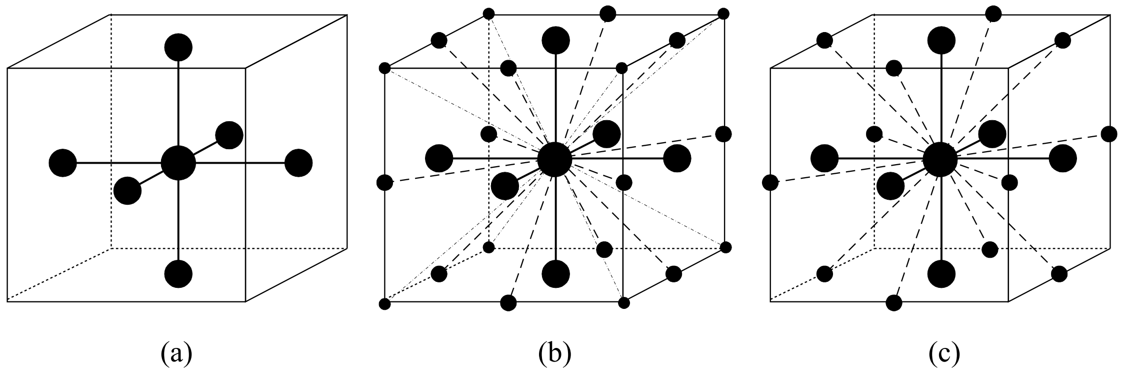

Fig. 1. Different versions of the digital waveguide mesh: a) the original rectangular mesh, b) the full interpolated structure with 27 connections, c) a sparse structure having 19 connections.

in which *p(n,x,y,z)* is the sound pressure in node *(x,y,z)* at time step *n*. There are a couple of ways to derive Eq. (1), and it can be done either by starting from the principles of digital waveguides as was done by van Duyne et. al. [1], or by discretizing the wave equation.

In a room acoustic simulation the space under study is divided into cubical elements, such that corners of the elements represent nodes of the mesh. Sound sources are placed in the mesh by forcing the sound pressures in corresponding nodes to desired values. The mesh has a certain update frequency $f_s = c\sqrt{3}/dx$, in which c is the speed of sound and dx is the distance between two neighboring nodes, i.e. the length of a side of a cube in the mesh. In each iteration of Eq. (1) the simulation is advanced by $dt = 1 / f_s$. For example, when $dx = 0.1$ m and $f_s \approx 5.9$ kHz then a simulation of 2 seconds requires $2 \times f_s \approx 12000$ simulation steps. The theoretical upper frequency limit is $f_s / 4$ since the magnitude response is aliased at that frequency.

There is one major problem in the original mesh structure, and it is the direction dependent dispersion. This dispersion error can be analyzed by the Von Neumann analysis that is based on spatial Fourier transform of Eq. (1) [4]. The details of the analysis are out of scope of this paper, and the reader is referred to previous articles [1,5]. As a result of the analysis a relative frequency error (RFE) is obtained.

The wave fronts propagate ideally in diagonal directions of the mesh, but in all the other directions there is dispersion. The situation is the worst in axial directions in which the wave propagation speed is decreased at the highest useful frequencies ($0.25 \times f_s$) as much as 24 % of the ideal speed. In practice this means that, e.g., the eigenfrequencies of a room can be significantly distorted. Therefore the valid frequency band of the original mesh is only a fraction of the theoretical maximum. For the frequency band $[0 \ldots f_s/10]$ the maximum relative frequency error is only 3.4%. The RFE curves for the original mesh are shown in Fig. 2a).

2.2 Interpolated 3-D Digital Waveguide Mesh

The reason for the direction dependent dispersion in the original rectangular mesh is caused by the fact that each node is connected only to its six axial neighbors. There are several techniques to overcome this problem. One way is to apply some non-rectangular mesh structure such as a tetrahedral mesh [6,7]. Another possible technique is the interpolated mesh structure [8,9,10]. The main advantages of the interpolated structure over the non-rectangular meshes are the ease of tessellation and simple implementation and for this reason we focus here only on the interpolated digital waveguide mesh.

In the interpolated structure nodes have connections also to 2-D diagonal and 3-D diagonal directions as illustrated in Fig. 1b). However, these connections are not of same length as the axial connections. To implement these connections fractional delays are applied such that in each non-axial connection the sound pressure is divided between the nodes that form the cube inside which the connection lies. Therefore the equation to govern this structure is a bit more complicated than Eq. (1):

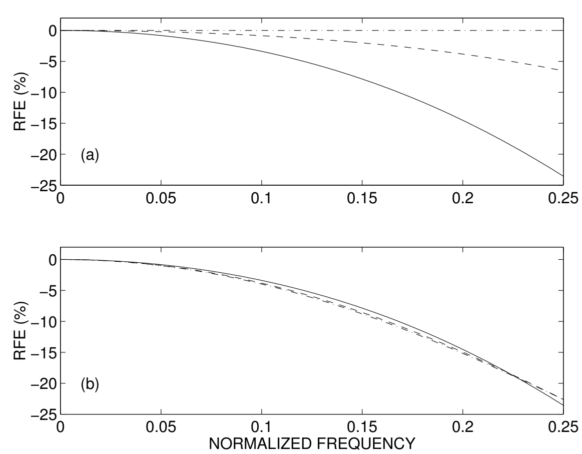

Fig. 2. Relative frequency error in the digital waveguide mesh a) in the original structure, and b) in the optimally interpolated mesh.

$$
\begin{aligned}
& p(n+1, x, y, z) \\
& = \sum_{l=-1}^{1} \sum_{m=-1}^{1} \sum_{p=-1}^{1} h(l, m, p) \quad p(n, x+l, y+m, z+p) \qquad (2) \\
& - p(n-1, x, y, z).
\end{aligned}
$$

Due to symmetry in the structure only four interpolation coefficients are required such that

$$
h(l, m, p) = \begin{cases} h_a, \text{ if } |l| + |m| + |p| = 1 \\ h_{2D}, \text{ if } |l| + |m| + |p| = 2 \\ h_{3D}, \text{ if } |l| + |m| + |p| = 3 \\ h_c, \text{ if } |l| + |m| + |p| = 0 \end{cases}. \qquad (3)
$$

There are various techniques to find out the interpolation coefficients, but in this case the best results have been obtained by numerically optimizing the coefficients. The goal in the optimization has been to reach wave propagation characteristics that are as uniform as possible in all directions. In Fig. 2(b) there are RFE curves for the optimally interpolated structure. The applied interpolation coefficients are listed in the following:

$$
\begin{aligned}
h_a &= 0.12052, \quad h_{2D} = 0.03860, \\
h_{3D} &= 0.01460, \quad h_c = 0.69686.
\end{aligned} \qquad (4)
$$

2.3 Sparse Mesh Structures

In the full interpolated structure each node has 27 connections: 6 axial, 12 2-D diagonal, and 8 3-D diagonal neighbors, and one delayed connection to the node itself. This number of connections makes the interpolated structure nearly five times heavier than the pure rectangular mesh from a computational point of view. For this reason we have investigated the possibility to reduce the number of connections by creating sparse structures in which some of the coefficients are set to zero. One efficient structure is achieved when $h_{3D} = 0$. It is nearly as accurate as the full interpolated structure but the computational load is reduced by one fourth. There are also other possible combinations that can be utilized, but they are less accurate, although they are even more efficient.

3 FREQUENCY WARPED DIGITAL WAVEGUIDE MESH

As can been seen from Fig. 2(b) there is still dispersion in the optimally interpolated mesh. Fortunately, the error is now nearly direction independent. This makes it possible to compensate the error up to certain degree. The technique we have applied for this is called frequency warping, and it is discussed in the following [11,12,4].

3.1 Frequency Warping

Frequency warping is a method to distort the frequency axis of a signal [13]. There are two principally different ways of warping a given signal, 1) time-domain and 2) frequency-domain warping. In practice, warping in the time-domain is accomplished using a chain of first-order digital allpass filters. The allpass filter parameter, or coefficient, will have the same value for all filter sections. The decision of this parameter, which is called the warping factor, determines how the frequency axis is warped. There will be a tap with a multiplying coefficient in the delay line between every two allpass filters. These coefficients will be assigned to the sample values of the signal to be warped. When a unit impulse, i.e., a digital signal consisting of a single '1' and an endless sequence of zero samples, is fed into the allpass filter chain, the output signal obtained as a sum of all tap outputs yields the warped version of the signal. Both the frequencies and the durations of all events in the signal are changed.

If the warping obtained with one pass through the allpass filter chain is insufficient for moving the frequencies where one wants them, a special trick must be used. It does not help to warp the signal multiple times with different warping factors, because any series of first-order warping operations is equivalent to a single warping with a certain factor. However, if the sampling rate of the signal is changed between consequtive warping stages, it is possible to obtain different warping functions than with single warping. In that case, the choice of the warping and sampling-rate-conversion factors for each stage becomes a non-trivial optimization problem, but the method becomes more flexible and powerful. We call this multiwarping [11].

Warping in the frequency domain refers to a process of resampling the spectrum of a signal and computing the corresponding temporal signal using the inverse discrete Fourier transform. This method is prone to numerical inaccuracies, because in general the frequency points where the new spectral samples should be read, are between the original spectral data points. High-resolution spectral computation with zero-padding and a long FFT is helpful, but additionally it may be necessary to use polynomial interpolation to determine the spectral values between the original points. An advantage of the frequency-domain

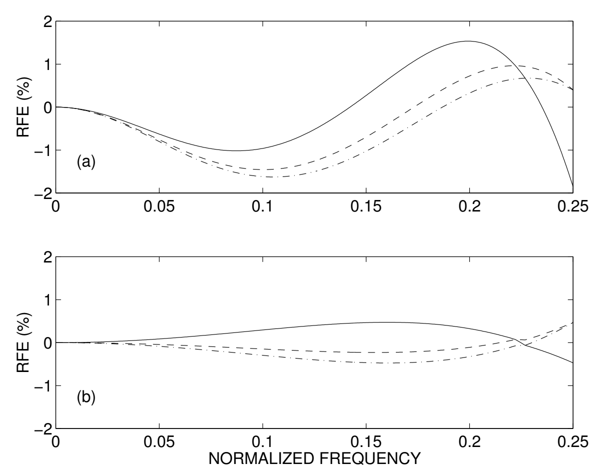

Fig. 3. Relative frequency error in the interpolated structure after frequency warpings: a) with two-stage multiwarping, and b) with warping in the frequency domain.

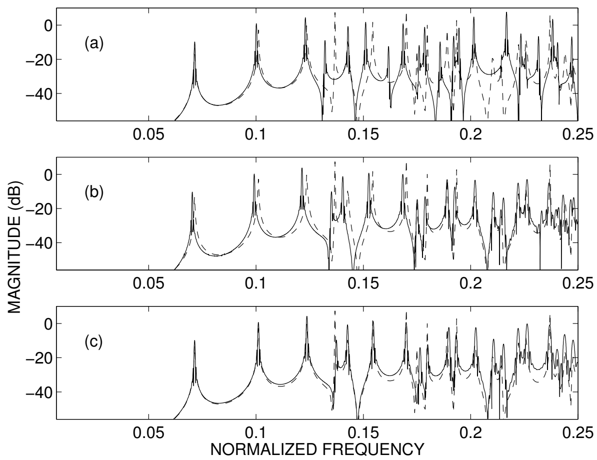

Fig. 4. A magnitude response of a cubic space calculated (a) with the original rectangular mesh, (b) with the optimally interpolated mesh applying multiwarping, and (c) with the optimally interpolated mesh using warping in the frequency domain. The solid line represents the simulation result and the dashed line is the analytical solution.

warping is that there are no principal restrictions for the warping function: any unique mapping of the frequency axis is possible.

3.2 Applying Frequency Warping with Digital Waveguide Mesh

Frequency warping is a process in which typically the whole signal is required before processing. Therefore it can be applied with digital waveguide mesh as a postprocessing operation. In practical simulations both the excitation signal and the simulation results should be frequency warped. The most accurate results can be obtained with warping in the frequency domain, but also the time-domain approach is possible. Figure 3a) presents RFEs in the optimally interpolated mesh with a two-stage multiwarping in the time domain, and in Fig. 3b) shows the RFEs after frequency-domain frequency warping. In the first case the maximal RFE is 2.0% and in the second one it is only 0.47%. For a typical room acoustic simulation this accuracy is enough.

4 SIMULATION RESULTS

This section presents two separate case studies. The goal of the first one is that the eigenfrequencies are correctly detected in a simple rectangular room that is easy to solve also analytically. The second case shows visually how diffraction is automatically included in the digital waveguide mesh method.

4.1 Case 1: Rectangular room

To show the accuracy of the digital waveguide mesh method a simple rectangular room ($8 \times 8 \times 8$=512 nodes) was both simulated and solved analytically. All the boundaries are ideal with reflection coefficient –1. The value is not physical in 3D room acoustics, but it was chosen since by that means fewer eigenmodes are visible in the magnitude response when compared to reflection coefficient 1. The source was located near one corner, and the receiver at the opposite corner. Figure 4a) shows the simulation result with the original structure. The results presented in Figs. 4b) and 4c) are obtained with the optimally interpolated structure applying coefficients shown in Eq. (4). In Fig. 4b) the result was frequency warped in the time domain, and in 4c) the warping was performed in the frequency domain. All the results are in good accordance with the theoretically obtained RFEs illustrated in Figs. 2a), 3a), and 3b), respectively. The frequencies shown in the figure are relative to the update frequency. If dx = 0.5 m the dimension of the room would be (4 m × 4 m × 4 m), and the relative frequency 0.25 would correspond to 300 Hz.

4.2 Case 2: Stagehouse

To study the edge diffraction we have made a simple model of a stage house. The sound source is located on the stage as illustrated in Fig. 5a). Figures 5b) and 5c) show two different visualizations corresponding to later time instants. The diffraction is most visible in the first order reflections from the sidewalls of the stage. Note the smooth attenuation of the reflected wavefront after the specular reflection part. Modeling of this phenomenon is difficult with the ray-based techniques, but in the wave-based methods it is inherent. Diffraction has crucial impact on geometrically complex spaces. One practical example is a typical opera house in which the orchestra is playing in the pit, and on the floor the audience hears only diffracted components of the direct sound.

5 BOUNDARY CONDITIONS

The development of the digital waveguide mesh has been going on for nearly ten years. At this point, the sound propagation in air is modeled quite efficiently and accurately. But this is not enough for practical room acoustic simulations. The wall materials have crucial affect on room acoustics, and therefore the boundary conditions of a mesh should correspond to real materials. Unfortunately, this issue has not yet been studied enough, and only some preliminary suggestions have been made [14], but a thorough survey on the topic is required. Currently, it is possible to have frequency independent reflection coefficients in a mesh, but only the coefficient value –1 is accurate. Thus all the physically relevant reflection coefficients in room acoustics are only approximations.

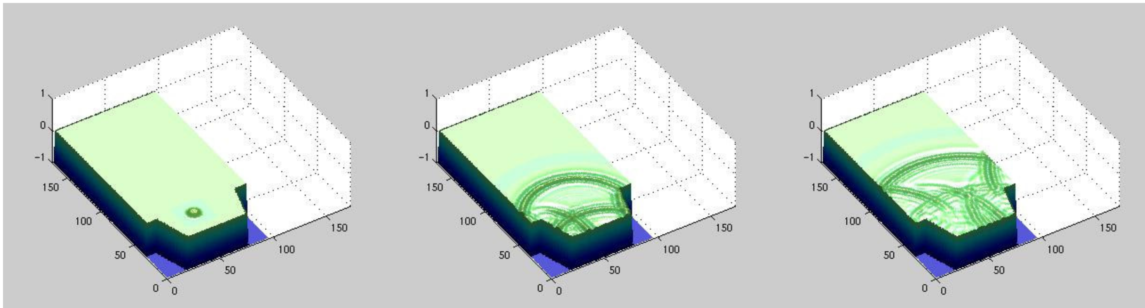

Fig. 5. Three visualizations of sound pressure in a stagehouse. Note especially the diffraction effects caused by the edges of the stage.

6 COMPUTATIONAL EFFICIENCY AND AURALIZATION

Current room acoustic auralization systems are typically based on geometrical acoustics, and therefore they are not accurate at the lowest frequencies. Digital waveguide mesh is one eligible candidate for this purpose since it is computationally efficient when compared to other wave-based techniques such as FEM (finite-element method) and BEM (boundary element method). Another advantage is that it is a time-domain technique, and the simulation results are easy to auralize.

Although the digital waveguide mesh is efficient, the computational load is still a severe problem, especially from a viewpoint of real-time auralization. Let us study an example with the pure rectangular mesh without interpolation. If we study a room of size 5 m × 10 m × 3 m with grid spacing of 0.2 m we have 25 × 50 × 15 = 18 750 nodes. For each node 19 additions and three multiplication, altogether 22 operations, are required. This means that each time sample takes 412 500 instructions. With 0.2 m grid spacing the sampling frequency will be 3 kHz thus resulting in 1237 MIPS (millions of instructions per second). The valid frequency range for auralizations depends on the application, but at most it is one fourth of the sampling rate. In this case the auralizations up to 750 Hz could be achieved with the given computational load. The load is still quite heavy, but computers get faster all the time, and in the future it should be possible to apply the technique in real-time at the low end of the frequency band. One advantage of the digital waveguide mesh is that the algorithm is easy to parallelize and thus it can gain benefit from multiprocessor computers [7].

7 CONCLUSIONS

The 3-D digital waveguide mesh is a wave-based method for room acoustic simulation. The method is a DSP oriented finite-difference time-domain technique. It has been under development for nearly ten years, and lots of the early problems have been solved. The direction-dependent dispersion of the original rectangular method is not a problem anymore since in the non-rectangular or interpolated rectangular mesh structures the direction dependence is negligible. The effect of remaining dispersion error can be remarkably reduced by frequency warping techniques. In the future, the most important research topic should be the boundary conditions. Currently, only simple wall materials can be simulated but to be a practical tool for room acoustic design frequency dependent reflection characteristics should be incorporated into the digital waveguide mesh method.

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